Assignment 01

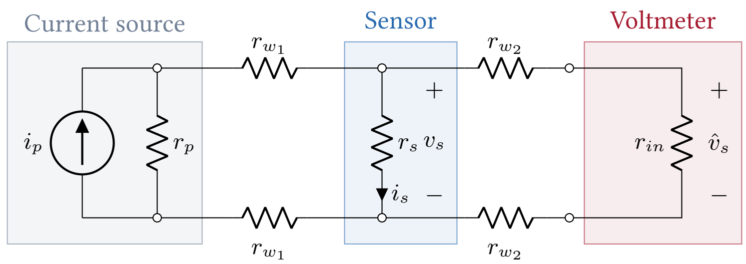
Transducers & Instrumentation · Module 1 — Measurement Fundamentals

Write out full answers to every question, with reasoning — not just the final result. A correct answer with no justification earns no marks.

1. In class we analysed a simplified sensor circuit: an ideal current source i_p driving a sensor resistance r_s, read by an ideal voltmeter. The open-circuit sensor voltage is $v_s = i_p r_s$, and a real voltmeter with input resistance r_{in} reads

$$\hat{v}_s = v_s \cdot \frac{r_{in}}{r_s + r_{in}}$$

Real circuits are messier. The figure below shows a more realistic model: the current source is non-ideal (an ideal source i_p in parallel with its internal resistance r_p), the connecting wires have finite resistance r_{w_1} and r_{w_2}, and the voltmeter has input resistance r_{in}.



(a) Derive an expression for $\hat{v}_s$ — the voltage read by the voltmeter — in terms of i_p, r_p, r_s, r_{w_1}, r_{w_2} and r_{in}.
Hint: replace all resistances with their series/parallel equivalents and apply the current-divider rule twice. [4]

(b) Show that your expression reduces to the ideal result $\hat{v}_s = v_s\, r_{in}/(r_s + r_{in})$ in the limits $r_p \to \infty$ and $r_{w_1}, r_{w_2} \to 0$. [2]

(c) A force sensor has $r_s = 15\,\Omega$ at the operating point of interest. The current source has $r_p = 500\,\Omega$, the wire resistances are $r_{w_1} = r_{w_2} = 0.5\,\Omega$, and the voltmeter has $r_{in} = 100\,\Omega$. Compute the percentage loading error

$$\text{error} = \frac{v_s - \hat{v}_s}{v_s} \times 100\%$$

Compare it with the error from the ideal two-element model (only r_s and r_{in}). Which parasitics matter most? [3]

(d) For the same sensor and wire resistances, what minimum r_{in} is needed to keep the total loading error below 1%? [3]

2. State the scale — **nominal**, **ordinal**, **interval**, or **ratio** — for each of the following, and justify your answer in one sentence.

(a) Calendar date of admission (e.g. 14 March 2026). [2]

(b) Length of hospital stay, in days. [2]

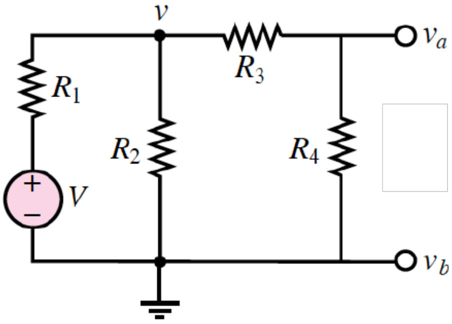
(c) Glasgow Coma Scale motor sub-score (1–6). [2]
The GCS grades depth of coma from three observed responses — eye opening, verbal, and motor — each scored separately. The motor sub-score runs from 1 (no movement) to 6 (obeys commands).

(d) Blood glucose concentration, in mg/dL. [2]

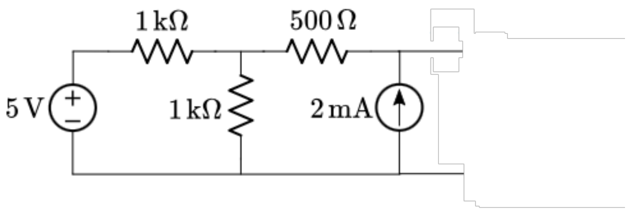
(e) Electrode–skin impedance, in kΩ. [2]

3. Draw the Thévenin and Norton equivalent circuits for a general electrical circuit. Explain how you would obtain each equivalent, and how you would convert between them. [4]

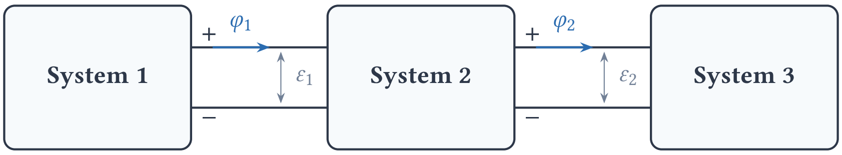
4. For the circuit below, find the Thévenin *and* Norton equivalents seen at the terminals v_a–v_b, for $V = 1\,\text{V}$ and $R_1 = 2\,\Omega,\ R_2 = 2\,\Omega,\ R_3 = 1\,\Omega,\ R_4 = 2\,\Omega$. [4]



5. For the circuit below, find the Thévenin *and* Norton equivalents seen at the output terminals on the right. [4]



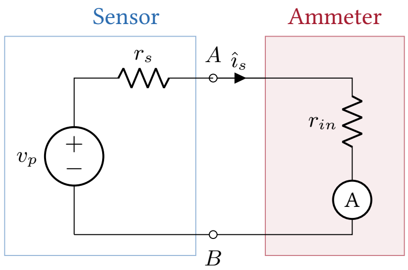
6. Three physical systems are connected in a chain, as shown below. System 2 has two ports: one facing System 1 and one facing System 3. The port variables are the effort ε and the flow φ marked at each port.



   (a) Write the power delivered by System 1 into System 2, and the power delivered by System 2 into System 3. [1]

   (b) Can the power delivered to System 3 ever *exceed* the power System 2 receives from System 1? Explain. Note that there may be other ports in this figure that are not shown. [3]

7. Consider a sensor whose output current is proportional to some measurand. The Thévenin equivalent circuit of the sensor is given in the following figure, and its output is connected to a real ammeter.



   (a) Derive the Norton equivalent circuit of the sensor. [1]

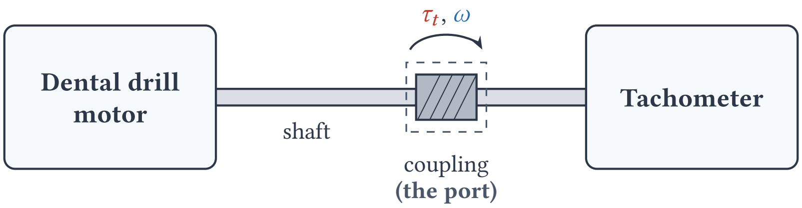
(b) Write down the expression for the current read by an ideal ammeter. [1]

(c) Write the current $\hat{\imath}_s$ read by the ammeter in variables associated with the Thévenin and Norton equivalent circuits for the sensor, and r_{in} of the ammeter. [2]

(d) What must r_{in} be for the reading to be within 1% of the true value? [1]

(e) How much power does the ammeter draw from the sensor? [1]

8. A dental drill motor is specified to run at a no-load speed of 40,000 rpm. Like any motor, its speed droops under load: this one loses 200 rpm for every 1 mN m of load torque applied to its shaft. You are asked to verify the speed specification using a *contact* tachometer, whose measuring wheel is pressed against the motor shaft and applies a drag (friction) torque of 2×10^{-3} mN m/rpm.



(a) Identify the effort and flow variables at the port between the motor and the tachometer. [1]

(b) What speed does the tachometer read? Express the error as a percentage, and state whether it reads high or low. [2]

(c) How much power does the tachometer draw from the motor at this speed? Comment on your answer. [2]

(d) Would a non-contact optical tachometer (reflective tape on the shaft, photodetector beside it) solve the problem? Explain in terms of the port variables. [1]

9. Complete the table of port variables, and write the expression for the power exchanged at a port. [2]

Domain	**Effort** ε	**Flow** φ
Electrical	________	________
Mechanical (translation)	________	________
Fluid	________	________

10. A sensor behaves as a Thévenin source with output resistance r_s. State the requirement on the input resistance r_{in} of (a) a **voltmeter** measuring its output voltage, and (b) an **ammeter** measuring its output current, without any loading in both cases. In one sentence, give the single rule that explains both. [2]

11. Define **precision** and **accuracy**, and state which type of error (random or systematic) degrades each. [2]

12. Consider two sensors for measuring pressure. The measurements from the two sensors are known to have the following models,

$$y_1 = y_{true} + N(3, 4) \qquad y_2 = y_{true} + N(0.5, 10)$$

where $N(\mu, \sigma^2)$ represents a normal or Gaussian distribution with mean μ and variance σ^2.

(a) Which of the two sensors is more accurate? [1]

(b) Which of the two sensors is more precise? [1]

(c) Now consider a third sensor with the model $y = y_{true} + N(0, 16)$. If you want the pressure measurement to have a standard error strictly less than 0.4 units, how many repeated measurements do you need? [2]

13. For N repeated measurements of the same measurand, write the expression for the **standard error** α. Then state the rule for how many significant figures to keep in the reported uncertainty, and how to round the mean to match. [2]

14. In static calibration:

    (a) Why must you wait before recording the output at each input level? [1]

    (b) Write the definition of the **residual** r_j obtained after fitting a function to the static calibration data. [1]

    (c) What is **hysteresis**? [1]

15. **Programming exercise — characterising an unknown sensor.** The module `sensor.pyc` contains one sensor. You cannot read its source code, and the only experiment it permits is evaluating its a **step** response: a response to a step input applied at $t = 0$ and held. The only feature you can change about the input is the step amplitude.

```python
import sensor

# a step of amplitude 1.0, held for 10 s, sampled every 10 ms
t, y = sensor.step_response(1.0, 10.0, 0.01)
```

The three arguments are the **amplitude** of the step, the **duration** in seconds over which the input is held, and the **sampling time** in seconds between consecutive readings. The call returns the time vector `t` and the sensor's readings `y`, both of the same length. The sensor is at rest before $t = 0$, and its readings are noisy. *You choose all three arguments* — and that choice, not the arithmetic that follows it, is what this question is really about.

(a) Is the sensor **static** or **dynamic**? State the experiment you ran, show the evidence, and provide your argument. [4]

(b) Is the sensor **linear** or **nonlinear**? A single step response cannot settle this — explain why. Then, design a set of step inputs, observe the output to ascertain if the system is linear. Provide your evidence and your argument. Note: *noise can make a linear sensor look nonlinear, if you use only a single measurement at each input level.* [3]

(c) Perform a full **static calibration** of the sensor. You must collect the data yourself. Justify each of your design choices: the range and spacing of the input levels, how long you held each level before recording, and how many readings you averaged at each level. You are encouraged to apply different inputs x_m covering a wide range, ideally repeating the same input multiple times, and record the output y. The typical operation of the system will be between $[-10, 10]$ for the input measurand. Tabulate your input and output data, and generate a scatter plot to visualize the relationship. Explain your findings about the static relationship. [5]

(d) Fit a polynomial function $y = f(x_m)$ to the static calibration data, and plot the fitted curve on the same axes as your scatter plot. Report the coefficients of the polynomial, and quantify the goodness of fit using the residuals. [3]

(e) The model of the fitted static relationship of the sensors is given by the following,

$$y_i = f\left(x_{m_i}\right) + e_i$$

where e_i is a random error term. Use your fitted polynomial to find an estimate of the measurement error. What would be a good measure of the size of the measurement error? Provide supporting arguments for your chosen measure of the size of the measurement error. [3]

(f) Find the **inverse** of your fitted function, i.e. the map $\hat{x}_m = f^{-1}(y)$ that recovers an estimate of the measurand from a sensor reading. Plot the true value of the measurand versus the estimated value of the measurand from the output values obtained from the static calibration data. Ideally, the points should lie on the $x = y$ line. Do they? What issues do you see? [3]

(g) Let's assume that this sensor is to be used to measure very slow varying or fixed measurands. You may therefore average N readings at a fixed input before inverting.

(i) From your estimate of the measurement error, find how many readings N must be averaged so that the uncertainty in $\hat{x}_m$ stays below 5% of the true measurand value. Plot N against x_m over the range $-10 \leq x_m \leq 10$, and identify the band over which a *single* reading is already good enough. [3]

(ii) To verify that your estimated N value over which you are averaging does provide you with the necessary error control, repeat the procedure (like the static calibration procedure), but repeatedly measuring the output N times for the same input, and averaging the estimates before reporting it. Confirm that your percentage error in your measurement procedure involving averaging over N value helps you keep the uncertainty to below 5%. [5]

Requires Python 3.10 with `numpy` *and* `scipy`. *Submit your code along with your answers; a plot without the code that produced it earns no marks.*

[Total: 29 marks]

Submission. A single PDF with your working shown for every question — an annotated circuit diagram where one is called for, full derivations, and numerical answers with a brief physical interpretation.

Total: 94 marks.